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Report on Research Training Group 2491

Fourier Analysis and Spectral Theory

First funding period

Speaker first funding period: Thomas Schick

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1 Research Programme

1.1 Central research idea

The RTG 2491 is dedicated to modern Fourier analysis and spectral theory. We take an interdisciplinary and innovative approach to this classical and powerful machinery and focus on its applications in the context of mathematical physics, topology and analytic number theory. Our concept is thematically and methodologically concentrated in an important area in analysis, which is investigated from a variety of different view points, each based on the profound expertise of the participating PIs.

A core theme of the RTG is analysis and spectral geometry on Riemannian manifolds, in particular, locally symmetric spaces or more generally spaces acted on by groups. Besides a topological structure, in many interesting cases studied they also have some arithmetic or combinatorial structure, and one of the key questions involves the fascinating interplay between the spectral properties of certain associated operators on the one hand, and geometric, topological, or arithmetic properties on the other. Some prototypical examples of this interaction featured in this RTG are analytic L^2-invariants, which link harmonic analysis to topology; and the resolvent and scattering theory of geometric differential operators on singular manifolds. Fourier and harmonic analysis also appear prominently in many applications of classical analytic number theory, in the representation theory of Lie groups and groupoids, and in the construction and investigation of quantum field theories with microlocal methods.

1.2 Major research achievements in the first funding period

The report focuses on the research results of the doctoral researchers of the first cohort (in finished and soon to be finished dissertations) and of the postdoctoral researchers.

The research carried out is grouped according to the following research areas:

(A) **Microlocal analysis and quantum field theory**

(B) **Spectral theory and harmonic analysis, analytic L^2-invariants**

(C) **Representations of Lie algebroids and Lie groupoids**

(D) **Geometric applications of Fourier theory**

(E) **Arithmetic applications of harmonic analysis**

A few doctoral students could have been classified in more than one of these topics.

Important methods that are used in several of these areas are microlocal analysis, symbolic calculi, trace formulas and Plancherel theory, Fourier analysis in numerous variations, spectral and scattering theory of operators, and C*-algebra techniques and representation theory. These common tools were highlighted in a number of lectures and informal reading courses. Due to the joint activities of the RTG, doctoral researchers in different topics were perfectly aware of this joint underlying methodology. They organised and participated in numerous discussions and joint learning activities (e.g. reading courses and seminars) and started collaborating across these topics.

The RTG continues to deepen the mutual flow of ideas within the triangle of analysis, topology, and analytic number theory.

During the first funding period, our guiding principle has been very useful and stimulating with respect to interaction between the different mathematical areas involved. This was fostered by joint meetings, lectures, and research talks, as well as the workshops and summer schools of the RTG. Well beyond the responsibilities of the thesis committees, there has been close collaboration and interaction between doctoral researchers, postdoctoral researchers and PIs.

(A) Microlocal analysis and quantisation Microlocal analysis is a powerful modern analytical tool. It helps to systematically treat PDEs of very different types, to construct calculi of pseudodifferential operators, and to investigate their structure. Since pseudodifferential calculi are a key ingredient in index theory, this is related to the index theory questions studied in project area (D). Microlocal techniques are also a key tool in the modern study of quantum field theory. One source of difficulties in quantum theory is that the most important operators are unbounded, but the mathematical theory needs bounded operators. Therefore, we have also studied the interplay between algebras of bounded and unbounded operators relevant in quantum theory.

The algebra of pseudodifferential operators on a manifold allows canonical transformations as extra symmetries. This allows one to build algebras of Fourier integral operators (FIOs), which consist of operators of the form $D = \sum_{g\in G} A_g\Phi_g$, where the A_g are (classical) pseudodifferential operators, and the Φ_g are quantised canonical transformations, that is, $g \mapsto \Phi_g$ is a representation of G by FIOs in the Calkin algebra. The Hannover node of the RTG, with doctoral researcher **Alessandro Contini**, postdoc **Eske Ewert** and PI Schrohe, studied the structure of such algebras and index theory for its elements. This approach encompasses not only the classical index problem for (pseudo)differential operators, but also the Atiyah–Weinstein problem or the Bär–Strohmaier index problem for Dirac operators on globally hyperbolic spacetimes.

Schrohe, in joint work with Savin, was able to define localised analytic and algebraic indices. In a similar vein, an algebraic index theorem was proven by Gorokhovsky, de Klein, and Nest. The results, however, remained intangible; even the connection between the two results remained unclear. It was therefore decided to also consider more concrete situations, where the FIOs act on function spaces over $\mathbb{R}^n$. Particularly accessible examples are the Heisenberg–Weyl operators T_w, $w \in \mathbb{C}^n$, or the elements of the complex metaplectic group. Recall that the complex metaplectic group is generated by the elements $\exp(i\mathrm{op}^w q + i\phi)$, where $\mathrm{op}^w q$ is the Weyl quantisation of a homogeneous real quadratic form q on $T^*\mathbb{R}^n$ and $\phi \in \mathbb{R}$.

[ET8] and [ET9] studied the algebras generated by operators of the form AR_g and AT_wR_g, respectively, where A is a Shubin type pseudodifferential operator, T_w is a Heisenberg–Weyl operator, and R_g is a suitable lift of $g \in U(n)$. These algebras allow concrete computations, and we were able to derive an index formula in [ET8] and to compute the Connes–Moscovici cyclic cocycles for noncommutative tori of arbitrary dimension and noncommutative orbifolds [ET9].

The Shubin type pseudodifferential operators cannot be combined with many FIOs to form an algebra because of their very restrictive symbol estimates. In that respect, the algebra of scattering (or SG) pseudodifferential operators is a more promising candidate. But while it has long been known that conjugation by complex metaplectic operators defines automorphisms of the Shubin operators, it was unclear what the automorphisms of the SG pseudodifferential operators are. **Alessandro Contini**'s PhD project is devoted to this question. The problem turned out to be rather complex, mostly because one has three mutually intertwined symbol levels instead of one. Nonetheless, Contini managed to classify the (order preserving) automorphisms of the SG calculus as given by conjugation with appropriate FIOs. This is analogous to a classical result of Duistermaat and Singer about automorphisms of algebras of classical pseudodifferential operators on compact manifolds. The SG-calculus has also been studied by **Eske Ewert** as part of her postdoc work. She uses the tools she developed in her thesis, that is, building the C*-algebra of order-zero pseudodifferential operators as a generalised fixed point algebra.

Three doctoral researchers in Göttingen working in project area 3.8 studied certain classes of boundary value problems and apply microlocal techniques in these explicit settings. **Gisel Mattar** investigated Fourier distributions with complex phase as a sophisticated new tool. She was able to describe the principal symbol explicitly. Using the geometric optic approximation, she has also understood the propagation of singularities and the regularity of solutions of certain hyperbolic initial boundary value problems, namely, those that are weakly regular and admit surface wave solutions. **Ryhana Darwich** generalised and studied wave front sets for vector-valued distributions. She described the propagation of these sets in a case not yet treated in the literature and then applied her theory to a specific partial differential equation, namely, the linearised magneto-hydrodynamics equation. **David Correa Cardeno** unified several approaches to prove well-posedness and *a priori* energy estimates for hyperbolic boundary value problems of weakly regular real type. Postdoctoral researcher Christian **Jäh** studied in [AA20] non-diagonalisable hyperbolic systems with space-dependent principal part, establishing a representation formula of the Cauchy problem in terms of Fourier integral operators, which allows one to derive regularity and propagation of singularities.

Another project in the area 3.8 by **Zhipeng Yang** developed harmonic analysis on a class of nilpotent Lie groups, namely, the 2-step stratified ones (see [AF10, AF11]). He used the representation theory of the underlying groups and the associated non-commutative Fourier transform to construct a global calculus of pseudodifferential operators and to compute the heat kernel of the sub-Laplacian. Harmonic analysis on nilpotent groups has been a key tool in several other dissertation projects supported by the RTG, including those of **Ewert**, **Han**, and **Höpfner**, which are described below in areas (C) and (B), respectively.

Microlocal techniques have also been a key technique in the two projects in quantum field theory that were pursued by doctoral researchers in the RTG. **Arne Hofmann**'s project aims at putting renormalisation in quantum field theory into the language of Lagrangian distributions. He has obtained interesting partial results towards this very ambitious goal. In particular, he treats the Wick products, which constitute the so-called 0^{th} step in renormalisation, in a way that is consistent with what is done in perturbation theory in later stages of the theory, and he interprets recent work of Moretti, Viet Dang, and others in a unified context. **Fabrizio Zanello** has been using the Bogolubov map to study the (infinitely many) conserved charges of the two-dimensional sine-Gordon model in Minkowskian signature, which is one of the few constructible models in quantum field theory. He works in the framework of pAQFT, that is, without relying on a particular representation. He is about to prove that all these charges are renormalisable. Following an abstract scheme due to PI Bahns and Wrochna to check if properties of a classical theory are preserved under renormalisation, he is studying whether the conserved charges may be renormalised in such a way that they stay in involution. A recent paper by Fröb and Cadamuro takes up this idea, which Zanello presented at a workshop, but they only study the first of the infinitely many conserved charges. One can clearly expect that Zanello's work will significantly contribute to the field. Bahns obtained further results on the sine-Gordon model in [ET1], constructing local nets of von Neumann algebras. Von Neumann algebras and their analytic structure are also the basis of the research on L^2-invariants, which play a central role in the research programme of the RTG.

Gauge theories play a key role in physics: from a mathematical perspective, all fundamental forces appearing in nature are understood as gauge theories. As a consequence, the Standard Model, which accounts for all interactions among elementary particles, is also a gauge theory.

Mathematically, a gauge theory is a physical theory containing symmetries, which do not impact on the experimental results. This discrepancy between these symmetries existing on a mathematical level but being undetectable at the physical level creates problems when trying to quantise theories of this type.

To face this problem, physicists have developed the Batalin–Vilkovisky construction as a cohomological approach to still extract relevant physical information despite of the appearance of divergences in the direct integral computations.

On the other hand, more recently, Connes proposed a purely mathematical formalisation of the Standard Model that is based on spectral triples, a key idea in noncommutative geometry. His model combines a finite-dimensional spectral triple, accounting for the particle interactions, with the Dirac operator on spacetime, which describes the gravitational part of the theory.

So far, however, there is no analogue of the BV formalism for Connes' theory, which still lives on a classical level.

The extension of the BV formalism towards such noncommutative geometry gauge theories is the goal of the RTG associated postdoc **Roberta Iseppi**. So far, she has built a BV-formalism for the fundamental case of finite-dimensional spectral triples (see[AA18, AA19]). It remains challenging, however, to further extend the construction to include the gravitational contribution, that is the spectral geometry of the Dirac operator that is the basis of Connes' model, with the resolutions and cohomological constructions of the BV-formalism.

Some of the analytic problems in quantum field theory already show up in finite-dimensional models, and are much more accessible in this simpler setting. Geometric quantisation is a scheme that starts with a Poisson manifold, describing a classical physical system, and aims at a C*-algebra describing a quantum system. Two doctoral researchers in the RTG, **Geoffrey Desmond Busche** and **David Kern** have worked on questions related to this construction.

The most important observables, like position and energy, are unbounded operators. Therefore, it is usually much easier to write down a *-algebra of unbounded operators that describes a particular quantum system. The mathematical theory, however, requires C*-algebras, that is, *-algebras of bounded operators. Thus, understanding how to relate a C*-algebra to a *-algebra of possibly unbounded operators is of fundamental importance for quantum physics. A purely mathematical instance of this is the relationship between the group C*-algebra of a simply connected Lie group G and the universal enveloping algebra of its Lie algebra $\mathfrak{g}$. A famous theorem of Nelson characterises which representations of $\mathfrak{g}$ "integrate" to a representation of G, namely, they are the ones where a certain "Laplacian" in the enveloping algebra of $\mathfrak{g}$ acts by an essentially self-adjoint operator. The thesis of Geoffrey Desmond **Busche** (in progress) proves an analogue of this where the enveloping algebra of a Lie algebra is replaced by the algebra of differential operators on a manifold M. When M is simply connected, the analogue of the group C*-algebra of G is the C*-algebra of compact operators. The theorem then says that any "integrable" representation of the *-algebra of differential operators on M is a direct sum of copies of the standard representation on $L^2(M)$. If $M = \mathbb{R}$, then the algebra of all differential operators contains the classical CAR algebra, generated by the commutation relation $[p, q] = \mathrm{i}\hbar$, as a dense subalgebra. Thus the result above is related to von Neumann's classical theorem about representations of the canonical anticommutation relations.

Busche's statement links two candidates for a "quantisation" of the cotangent manifold T^*M. One is the *-algebra of differential operators on M, whose representations usually involve unbounded operators; the other is a C*-algebra, which is associated to a Lie groupoid. Its underlying Lie algebroid is, however, more flexible and closer to the original Poisson manifold, so that it may be built in more cases. In particular, one step in geometric quantisation is to reduce a Lie groupoid using a polarisation. In the example of T^*M, this step replaces T^*M by M. In general, however, it may only give a foliation instead of a fibration, and then it is unclear how to proceed. A foliation on a Lie algebroid that is compatible with the algebraic structure is called an infinitesimal ideal systems (IIS). **David Kern** has found a more general concept of a "reduction system", which may be used to descend a Lie algebroid structure to a quotient. He then went on to study analogues of this for higher Lie algebroids. The associated doctoral researcher **Ilias Ermeidis** has been studying the deformation theory of ordinary IIS both in Lie algebroids and in Lie groupoids. Following the paradigm of derived algebraic geometry, he describes this in a deformation complex with an L_∞-algebra structure. So far, he has also been able to show that an appropriate vanishing condition for his deformation cohomology implies stability and rigidity of an IIS.

(B) Spectral theory and harmonic analysis, analytic L^2-invariants A second core topic of the RTG uses classical harmonic analysis on symmetric spaces and Lie groups as well as work derived from this. We use the Plancherel decompositions to get explicit knowledge of the spectrum of the differential form Laplacian on non-linear semisimple Lie groups on the one hand, and on nilpotent Lie groups on the other. These tools are used to derive trace formulas that allow us to relate dynamical zeta functions to torsion invariants on hyperbolic manifolds of dimension 3. The specific invariants one wants to compute include L^2-invariants defined by using the regularised dimension function of an underlying von Neumann algebra. Here, the RTG research focused in particular on Novikov–Shubin invariants and the L^2-torsion. These were also investigated in new situations like anti-fractals. The L^2-invariants have a dynamical touch and we investigated how they are encoded in the return probabilities of new higher-dimensional random walks (going beyond the classical case of random walks on graphs to random walks on higher-dimensional cellular complexes).

The doctoral thesis of **Zhicheng Han** achieves the complete explicit calculation of the spectrum of the Laplace–Beltrami operators and the geometric Dirac operator on the universal cover of $\mathrm{Sl}_2(\mathbb{R})$, which is a non-linear Lie group. This is used to compute all Novikov–Shubin invariants of this space (and of compact quotients, like the unit circle bundle of compact hyperbolic surfaces). These calculations confirm a conjecture of Gromov on the invariance of Novikov–Shubin invariants under measure equivalence for the specific example of the unit circle bundle and the trivial circle bundle over hyperbolic surfaces. Han's thesis also contains significant foundational work for arbitrary symmetric spaces, including the derivation of explicit formulas for the differential form Laplacian via Plancherel theory.

Doctoral researcher **Tim Höpfner** constructed in one of three sub-projects of his thesis an appropriate random walk on the k-cells of the universal covering of a finite CW-complex X. He related the decay of its return probabilities to the Novikov–Shubin invariant in degree k of the complex X. In a second sub-project, Höpfner introduced a two-parameter version of Novikov–Shubin invariants for bundles of manifolds. This allowed him to measure the interplay between the spectral theory of the base and the fibre in the spectrum of the Laplace operator. He derived fundamental properties like homotopy invariance, and carries out explicit computations in first examples, in particular, for the Heisenberg group, written as an extension with 1-dimensional centre (the fibre) and two-dimensional quotient (the base). Finally, a new method to get information on the Novikov–Shubin invariants of differential form Laplace operators on nilpotent Lie groups is derived. Here, Höpfner used the Chevalley–Eilenberg complex to achieve explicit calculations in new cases. Alternative routes to this use Plancherel decompositions and the general foundations worked out by **Han**; this also connects to the global pseudodifferential calculus constructed in the thesis of **Yang**.

The associated researcher **Engelbert Suchla** has constructed in his thesis [AA26] L^2-invariants like L^2-Betti numbers, Novikov–Shubin invariants, and L^2-torsion for large-scale self-similar spaces, for example, anti-fractals as introduced by Guido and Isola. He derives Künneth formulas for these invariants and proves their homotopy invariance under an appropriate notion of homotopy. The most significant and most recent results are obtained for L^2-torsion. He relates his new invariants to the spectral theory of random walks on these anti-fractals, which was studied previously by Woess and others.

The RTG funded postdoctoral researcher **Léo Bénard** studied in particular various types of analytic torsion for hyperbolic 3-manifolds. In a fundamental paper with Dubois, Heusener, and Porti he shows that the asymptotics of the twisted Alexander polynomials (a twisted variant of analytic torsion) behaves like the exponential of the hyperbolic volume [ET2]. Further results study torsion invariants as functions on the character variety of 3-manifolds [AF5, AF4, AF3] and prove interesting properties. As one example, [AF3] establishes twisted L^2-torsion as a function on the character variety of a 3-manifold and shows, in particular, that it is a real analytic function around the holonomy representation of a hyperbolic 3-manifold.

The Fried conjecture relates the analytic torsion of a 3-manifold to the (dynamical) Ruelle zeta function, stating in favourable situations that the former equals the value of the latter at zero. The very recent work [PF1] (with ongoing follow-up projects with the recently hired RTG postdoc **Polyxeni Spilioti** and Frahm) proves a conjecture of Freed for the unit circle bundle of hyperbolic surfaces with singularities and twists by finite-dimensional linear representations of the fundamental group.

(C) Index theory and groupoids Lie groupoids were first used in index theory by Connes to give an alternative proof of the Atiyah–Singer Index Theorem for elliptic pseudodifferential operators. For hypoelliptic (pseudo)differential operators or on manifolds with singularities, Lie groupoids have become even more crucial. In many cases, the symbol algebras themselves become noncommutative, which makes it more challenging to define a "topological" index.

A crucial ingredient – both to state and prove an index theorem – is a pseudodifferential calculus and, in particular, a C*-algebra of order-zero pseudodifferential operators. Researchers trying to do index theory for hypoelliptic operators have realised for some time that the relevant pseudodifferential calculus is closely related to an appropriate generalisation of Connes' adiabatic tangent groupoid. This relationship was made precise in **Eske Ewert**'s dissertation and her follow-up publications [PF3, PF4, AF7]. She described the C*-algebra of order-zero pseudodifferential operators as the generalised fixed point algebra for the scaling action on a certain ideal in the groupoid C*-algebra of the relevant groupoid. She used intrinsic properties of this alternative construction to clarify some features of this C*-algebra that are useful for index theory. She worked right away in the setting of a filtered manifold, which is relevant for many hypoelliptic index problems. The important special case of the pseudodifferential calculus on a nilpotent Lie group was treated first. Her ansatz is flexible enough to also work for other index problems. She is now applying this method to the Fourier integral operators mentioned above under (A). Groupoid methods to build pseudodifferential calculi are also used by the associated postdoctoral researcher **Vito Zenobi** in [AA31].

Hörmander found a very powerful sufficient condition for a second-order differential operator to be hypoelliptic. If it is written in local coordinates as $f + X_0 + \sum_{j=1}^{k} X_j^2$ with vector fields $X_0, \dots, X_k$, then it suffices if iterated brackets of these vector fields span the tangent space at each point. This produces a filtration on vector fields, which often comes from a filtered manifold structure. In general, however, the relevant subbundles in the filtration may be singular, that is, their fibre dimensions may jump. It is still unclear how to do index theory in the presence of such singularities.

A related case where index theory has already been developed by Androulidakis and Skandalis is the case of elliptic longitudinal operators for a singular foliation. An ordinary foliation gives rise to a groupoid, its holonomy groupoid. In contrast, a singular foliation should give a higher groupoid. This is because the arrow space of Androulidakis' holonomy groupoid is still a very singular object. It should, therefore, be written again as the orbit space of a groupoid, which may again be singular. On the infinitesimal level, Lavau in 2016 defined a higher Lie algebroid from suitable singular foliations. The postdoc **Leonid Ryvkin** associated to the RTG has studied the geometric and algebraic features of higher Lie algebroids and Lie groupoids, with a particular eye on the case of singular foliations (see [AA22, AA23, AA24, AA25, PA12, PA13]). In particular, he proved that the tangent functor for n-groupoids is representable.

The gap between index theory applications and higher Lie groupoids still seems wide, but we believe that it will be closed by future research. An ordinary Lie group(oid) is turned into a higher one by the nerve construction. The geometric realisation of the nerve gives the classifying space, which in turn produces characteristic classes. Thus building a higher Lie groupoid from a singular foliation is roughly the same as building its "classifying space" and a prerequisite for studying its characteristic classes. Both classifying spaces and characteristic classes proved important in better understood index problems.

In a more down-to-earth setting, characteristic classes are studied by the associated doctoral researcher **Rosa Marchesini**. In her dissertation, she proves generalisations of the Bott vanishing theorem for Lie algebroids and that certain classes in Lie algebroid cohomology are invariant under suitable homotopies. Her work also helps to explain the obstructions to the existence of infinitesimal ideal systems found by Jotz Lean.

Shifted symplectic higher groupoids are a powerful tool for studying the geometry of higher spaces. They are an extension of the usual symplectic groupoids, an important concept in geometric quantisation. Shifted symplectic higher groupoids generalise the concept of a symplectic groupoid to higher groupoids up to homotopy, allowing for the study of more complex geometric structures. **Miquel Cueca Ten**'s work studied the basic concept of a shifted symplectic higher groupoid, as well as its applications in geometry. He also looked at some examples of shifted symplectic higher groupoids, including the classifying stack BG, which is 2-shifted symplectic. Together with PI Zhu, this was achieved in [ET5].

Returning to ordinary groupoids, they are also useful to study or even define important classes of C*-algebras. The most relevant groupoids here are the étale ones, however, which have a trivial Lie algebroid. The realisation that the Cuntz C*-algebras are groupoid C*-algebras was one of the motivations to develop their theory in the 1980s. Large classes of combinatorially defined C*-algebras, such as the C*-algebras of higher-rank, topological or self-similar graphs, are also groupoid C*-algebras. The associated doctoral researcher **Celso Antunes** studies a general method to build these groupoids and thus their C*-algebras, starting with a "self-correspondence" on an étale groupoid. He generalises previous work by Albandik, a former doctoral student of PI Meyer. Some basic results about groupoid correspondences and their bicategorical structure were published by Antunes, Ko and PI Meyer in [AA1].

Renault's Cartan subalgebra theorem characterises when a C*-algebra is a groupoid C*-algebra. It offers a good explanation why so many important C*-algebras have this structure. One of the most important examples of a self-similar group, the Grigorchuk group, leads to a groupoid whose arrow space fails to be Hausdorff. This and other similar examples have motivated C*-algebraists to extend various results to non-Hausdorff étale groupoids. Kwaśniewski and PI Meyer have recently proven results about the ideal structure of groupoid C*-algebras that also cover the case of non-Hausdorff groupoids [ET7]. The associated researcher **Jonathan Taylor** has extended Renault's Cartan subalgebra theorem to the non-Hausdorff case and also studied a noncommutative version of Renault's Cartan subalgebras due to Exel (see [PA14, PA15, PA16]).

(D) Geometric applications of Fourier theory The tools developed in the RTG have strong applications to the study of the geometry of smooth manifolds, in particular, by applying index theory, which brings in the geometry through geometrically defined differential operators (like the Dirac operator). The most powerful such invariants are "higher" invariants that also combine ideas from non-commutative geometry, in particular, K-theory of C*-algebras associated to the situation. To pass to computable invariants, one combines this with (non-commutative) homological tools which are also investigated in topic (A) by PI Schrohe and his group. A very systematic study of this has been carried out in a series of papers by Higson and Roe under the title "mapping surgery to analysis", later complemented by Piazza and PI Schick to take more geometric information into account. The step to pass to homology was missing and was a major result of the joint work of associated postdoc **Vito Zenobi**, PI Schick and Piazza [ET6]. This used Zenobi's groupoid approach to pseudodifferential calculi [AA31], which was also used for geometric applications in [AA33]. The work [ET6] has concrete applications to the question of whether a given smooth manifold admits a Riemannian metric of positive scalar curvature, and how to understand the space of such metrics; compare [AA32] and also, in particular, the joint paper with associated RTG postdoc **Simone Cecchini** and with former Göttingen master student Seyedhosseini [PA1].

The geometry of positive scalar curvature has been a focus of the RTG's geometric research programme beyond the qualitative results we just described. A more sophisticated research direction initiated by Gromov asks for fine quantitative measures of the possible content of positive scalar curvature, like the neck problem or the cylinder problem: how long can a neck on a smooth manifold with complicated boundary or the cylinder over a complicated manifold be such that the total manifold still carries metrics with scalar curvature bounded below by a fixed positive constant (and with convex boundary). This is studied with the help of appropriate boundary conditions and the associated index theory. Postdoc **Cecchini**, partly joint with RTG replacement professor **Rudolf Zeidler** have obtained optimal results combining in sophisticated ways Dirac operators with a potential and pushed these methods into the realm of higher index theory via C*-algebras and their K-theory [ET4, AA4, AA3].

The doctoral research of associated member **Jialong Deng** also centred around these questions and ideas. He focused on more topological aspects and the effects of low regularity metrics (also generalising partly beyond the case of manifolds). His doctoral research led to a series of publications that clarified the relations between different approaches to positive curvature [AA13] (via optimal transport and infinitesimal growth of balls) and proved new rigidity theorems in positive (scalar) curvature [AA10, AA11, AA12].

The research line around index theory and geometric applications is complemented by the work of postdoctoral researcher **Christopher Wulff** who focused on the index theory and C*-algebra K-theory of large-scale geometry. This area is just becoming a more prominent part of the RTG through the research programme of newly included PI Vigolo. Wulff's main focus was to develop structures in coarse K-theory like cup and cap products [AA27, AA29, AA30, PA17] with applications to the calculation of coarse indices of Dirac operators. This was then also used to construct obstructions to the existence of metrics of positive scalar curvature [AA28] (joint with RTG replacement professor **Zeidler**).

Besides the very explicit use of index theory and K-theory for geometric questions, another aspect in this research topic was the investigation of fundamental properties of K-theory on interesting spaces.

The doctoral project of **Anne Prepeneit** studies the equivariant K-theory of spaces with a smooth action of a p-adic group $\mathsf{Sl}_n(\mathbb{Q}_p)$ or, more generally, of a totally disconnected group.

Smooth here means that all stabiliser groups are compact and open. Prototypical examples are the action of $\mathrm{Sl}_n(\mathbb{Q}_p)$ on its Bruhat–Tits building, with its significant arithmetic properties. She gives a new homotopy theoretic model for this, in terms of a classifying space that is a Banach Lie group and carries an explicit Chern character differential form. This allows her to construct a Chern character isomorphism very explicitly and to prove that this is rationally an isomorphism, where the target involves the extended quotient of Baum and Connes.

In a similar, but more C*-algebraic direction, associated doctoral researcher **Tom Dove** studies twisted equivariant K-theory, now primarily for actions of finite groups and compact Lie groups. The equivariant K-theory is defined in terms of C*-algebra associated to the twist. The theory comes up prominently in the study of equivariant T-duality, a relation inspired by type (A) and type (B) models of QFT. Joint with RTG visitor **Velásquez** and PI Schick, he proved an Atiyah-Segal type decomposition theorem for twisted equivariant K-theory [PA6] and a general local description of T-duality for higher dimensional fibres [PA5]. He has by now proven that the canonical T-duality transformation is indeed an isomorphism in twisted equivariant K-theory for actions by arbitrary compact Lie groups using a good understanding of the representation theoretic Fourier decompositions.

(E) Arithmetic applications of harmonic analysis The RTG also studies geometric situations from an arithmetic point of view. In particular, doctoral researcher **Florian Munkelt** counts the asymptotics of rational points close to a given submanifold of Euclidean space under relaxed non-degeneracy conditions on the submanifold and its curvature, improving results of Huang, Schindler, Yamagishi. This has already resulted in a first preprint [PF6]. More concretely, given a projective variety in $\mathbb{P}^n_{\mathbb{Q}}$, one asks about the number of rational points of bounded height on such varieties. In the case of Fano varieties, this is related to questions studied by doctoral students **Bernert** and **Havlas** on Arithmetic Fourier analysis. Complementary to those projects, which focus on very specific classes of Fano varieties, Munkelt used as little information as possible and still produced non-trivial upper bounds. This is related to the active area of the dimension growth conjecture, which a few years ago saw a breakthrough through the work of Huang, who managed to prove the dimension growth conjecture for certain hypersurfaces in projective space using only methods from harmonic analysis. These results have further been generalised by PI Schindler and Yamagishi to manifolds of higher codimension under certain curvature conditions [ET10], again with purely analytic methods. Munkelt has further developed this line of research by relaxing these curvature conditions.

The more classical, diophantine geometric aspects of research topic (E) studies rational points on algebraic varieties. This is a general field with numerous well-established principles and questions. The problems studied in the RTG centre around varieties of low degree (quadratics, cubics, etc.). Driving principles are, among others, the validity of the Hasse principle and Manin's conjecture, and the question of for which minimal number of variables one can prove these results. Among the fundamental tools are modern variants of the Hardy–Littlewood circle method.

PI Schindler has worked in collaboration with Browning and Pierce on the Hasse principle and density of rational points of systems of quadratic equations over the rational numbers.

Motivated by results on Weil restrictions of a single quadratic form over a number field, this collaboration established first a bijection between certain systems of quadratic equations over the rational numbers and certain generalised quadratic forms over number fields. These were then treated with the circle method, more precisely the delta method. This has lead to new examples of families of intersections of quadrics for which they could establish the Hasse principle and Manin's conjecture.

Doctoral researcher **Rok Havlas** refines the circle method to prove the Hasse principle for quintic hypersurfaces. In his work, he improved bounds on the number of variables from the results of Birch, Browning and Heath-Brown. This is achieved by refining a certain differencing process, as well as numerical optimisations with the aid of a computer.

The work of doctoral researcher **Christian Bernert** studies the corresponding problem for cubic equations. He revisited Davenport's approach via the geometry of numbers, replaces it with an elementary argument and renewed the treatment of the local aspects entirely. As a result one now has perfect control on the singular series in all dimensions that matter here. In joint work with Hochfilzer, this has been extended to imaginary quadratic number fields, with applications to intersections of two cubic hypersurfaces (see below).

Doctoral researcher **Leonhard Hochfilzer** focused on arithmetic geometry beyond the field of rational numbers. He established (among other results) bounds on the asymptotics of rational points on diagonal cubic forms in six variables in finite characteristic, answering a 1964 question of Davenport [PF5]. In a different direction, he completed the proof of Artin's primitive root conjecture over function fields. With fellow doctoral researcher **Bernert** he shows that cubic forms in at least 14 variables over imaginary quadratic number fields always have a solution. As a consequence, every cubic hypersurface in at least 31 variables contains rational lines.

Considerable progress has been made to control "arithmetically defined Fourier coefficients", a core theme of arithmetic harmonic analysis. In joint work with Wooley, PI Brüdern combined an older combinatorial approach with the Fourier moment technique that was found by the same authors in 2015. This produced new insight in intersections of diagonal quartic hypersurfaces. In recent work, a new "major arc moment estimate" was proposed by Brüdern and Wooley. This paved the way to new world records for Waring's problem and related questions in additive number theory [ET3]. The consequences for the Fourier moment technique are yet to be explored.

2 Publications

2.1 Publications by doctoral and postdoctoral researchers of the RTG

The names of the RTG researchers are marked by typesetting them boldface.

2.1.1 Peer-reviewed publications of doctoral and postdoctoral researchers who received funding from the RTG

[AF1] Bernd Ammann, Jérémy **Mougel**, and Victor Nistor, *A comparison of the Georgescu and Vasy spaces associated to the N-body problems and applications*, Ann. Henri Poincaré **23** (2022), no. 4, 1141–1203, DOI 10.1007/s00023-021-01109-1.

[AF2] Léo **Bénard**, Jérôme Dubois, Michael Heusener, and Joan Porti, *Asymptotics of twisted Alexander polynomials and hyperbolic volume*, Indiana Univ. Math. J. **71** (2022), no. 3, 1155–1207.

[AF3] Léo **Bénard** and Jean Raimbault, *Twisted L^2-torsion on the character variety*, Publ. Mat. **66** (2022), no. 2, 857–881, DOI 10.5565/publmat6622211.

[AF4] Léo **Bénard**, *Torsion function on character varieties*, Osaka J. Math. **58** (2021), no. 2, 291–318.

[AF5] ______, *Reidemeister torsion form on character varieties*, Algebr. Geom. Topol. **20** (2020), no. 6, 2821–2884, DOI 10.2140/agt.2020.20.2821.

[AF6] Léo **Bénard** and Anthony Conway, *A multivariable Casson-Lin type invariant*, Ann. Inst. Fourier (Grenoble) **70** (2020), no. 3, 1029–1084 (English, with English and French summaries).

[AF7] Eske Ellen **Ewert**, *Index theory and groupoids for filtered manifolds*, PhD thesis, Universität Göttingen, 2020.

[AF8] Eske Ellen **Ewert** and Ralf **Meyer**, *Coarse geometry and topological phases*, Comm. Math. Phys. **366** (2019), no. 3, 1069–1098, DOI 10.1007/s00220-019-03303-z.

[AF9] Jan Frahm and Polyxeni **Spilioti**, *Twisted Ruelle zeta function at zero for compact hyperbolic surfaces*, J. Number Theory **243** (2023), 38–61, DOI 10.1016/j.jnt.2022.08.003.

[AF10] Zhipeng **Yang**, *Harmonic analysis on 2-step stratified Lie groups without the Moore-Wolf condition*, Doctoral theis, Universität Göttingen, 2022.

[AF11] ______, *A note on the heat kernel for the rescaled harmonic oscillator from two step nilpotent Lie groups*, Acta Math. Sin. (Engl. Ser.) **38** (2022), no. 9, 1597–1611, DOI 10.1007/s10114-022-2100-8.

[AF12] Yongpeng Chen and Zhipeng **Yang**, *Existence and asymptotical behavior of multiple solutions for the critical Choquard equation*, J. Geom. Anal. **32** (2022), no. 9, Paper No. 238, 34, DOI 10.1007/s12220-022-00976-2.

[AF13] Guangze Gu and Zhipeng **Yang**, *On the singularly perturbation fractional Kirchhoff equations: critical case*, Adv. Nonlinear Anal. **11** (2022), no. 1, 1097–1116, DOI 10.1515/anona-2022-0234.

[AF14] Zhipeng **Yang**, *Non-degeneracy of positive solutions for fractional Kirchhoff problems: high dimensional cases*, J. Geom. Anal. **32** (2022), no. 4, Paper No. 139, 24, DOI 10.1007/s12220-022-00880-9.

[AF15] Guangze Gu and Zhipeng **Yang**, *Positive eigenfunctions of a class of fractional Schrödinger operator with a potential well*, Differential Integral Equations **35** (2022), no. 1-2, 123–150.

[AF16] Zhipeng **Yang** and Yuanyang Yu, *Existence and concentration of solution for Schrödinger-Poisson system with local potential*, Partial Differ. Equ. Appl. **2** (2021), no. 4, Paper No. 47, 22, DOI 10.1007/s42985-021-00105-8.

[AF17] Zhipeng **Yang**, *Fujita exponent and nonexistence result for the Rockland heat equation*, Appl. Math. Lett. **121** (2021), Paper No. 107386, 6, DOI 10.1016/j.aml.2021.107386.

[AF18] Zhipeng **Yang** and Fukun Zhao, *Multiplicity and concentration behaviour of solutions for a fractional Choquard equation with critical growth*, Adv. Nonlinear Anal. **10** (2021), no. 1, 732–774, DOI 10.1515/anona-2020-0151.

[AF19] Zhipeng **Yang** and Yuanyang Yu, *Geometrically distinct solutions of nonlinear elliptic systems with periodic potentials*, Arch. Math. (Basel) **115** (2020), no. 6, 703–716, DOI 10.1007/s00013-020-01519-3.

[AF20] Xianyong Yang and Zhipeng **Yang**, *Existence of multiple non-trivial solutions for a nonlocal problem*, AIMS Math. **4** (2019), no. 2, 299–307, DOI 10.3934/math.2018.2.299.

[AF21] Zhipeng **Yang**, Yuanyang Yu, and Fukun Zhao, *The concentration behavior of ground state solutions for a critical fractional Schrödinger-Poisson system*, Math. Nachr. **292** (2019), no. 8, 1837–1868, DOI 10.1002/mana.201700398.

[AF22] Marcella Palese, Olga Rossi, and Fabrizio **Zanello**, *Geometric integration by parts and Lepage equivalents*, Differential Geom. Appl. **81** (2022), Paper No. 101866, 26, DOI 10.1016/j.difgeo.2022.101866.

2.1.2 Publications available as preprint of doctoral and postdoctoral researchers who received funding from the RTG

[PF1] Léo **Bénard**, Jens Frahm, and Polyxeni Spilioti, *The twisted Ruelle zeta function on compact hyperbolic orbisurfaces and Reidemeister-Turaev torsion*, 2021. arXiv:2110.06683.

[PF2] Léo **Bénard**, Vincent Florens, and Adrien Rodau, *Slope invariant and the A-polynomial of knots*, 2021. arXiv:2103.14151.

[PF3] Eske Ellen **Ewert**, *Pseudodifferential operators on filtered manifolds as generalized fixed points*, 2021. arXiv:2110.03548, submitted to Journal of Noncommutative Geometry.

[PF4] ———, *Pseudo-differential extension for graded nilpotent Lie groups*, 2020. arXiv:2002.01875.

[PF5] Leonhard **Hochfilzer** and J. Glas, *On a question of Davenport and diagonal cubic forms over $\mathbb{F}_q(t)$*, 2022. arXiv:2208.05422.

[PF6] Florian **Munkelt**, *On the number of rational points close to a compact manifold under a less restrictive curvature condition*, 2022. arXiv:2205.06183.

[PF7] Bernd Ammann, Jérémy **Mougel**, and Victor Nistor, *A regularity result for the bound states of N-body Schrödinger operators: Blow-ups and Lie manifolds*, 2022. arXiv:2012.13902.

[PF8] Polyxeni **Spilioti** and Frédéric Naud, *On the spectrum of twisted Laplacians and the Teichmüller representation*, 2022. arXiv:2205.09540.

2.1.3 Peer-reviewed publications of doctoral and postdoctoral researchers who received funding from other sources

[AA1] Celso **Antunes**, Joanna Ko, and Ralf **Meyer**, *The bicategory of groupoid correspondences*, New York J. Math. **28** (2022), 1329–1364, DOI 10.1177/1077546321993331.

[AA2] Simone **Cecchini** and Thomas **Schick**, *Enlargeable metrics on nonspin manifolds*, Proc. Amer. Math. Soc. **149** (2021), no. 5, 2199–2211, DOI 10.1090/proc/14706.

[AA3] Simone **Cecchini**, *A long neck principle for Riemannian spin manifolds with positive scalar curvature*, Geom. Funct. Anal. **30** (2020), no. 5, 1183–1223, DOI 10.1007/s00039-020-00545-1.

[AA4] ———, *Callias-type operators in C^*-algebras and positive scalar curvature on noncompact manifolds*, J. Topol. Anal. **12** (2020), no. 4, 897–939, DOI 10.1142/S1793525319500687.

[AA5] Simone **Cecchini** and Rudolf **Zeidler**, *Scalar and mean curvature comparison via the Dirac operator*, Geometry & Topology, DOI 10.48550/arXiv.2103.06833. compare G&T list of upcoming articles.

[AA6] Miquel **Cueca** and Chenchang **Zhu**, *Shifted symplectic higher Lie groupoids and classifying spaces*, Adv. Math. **413** (2023), Paper No. 108829, DOI 10.1016/j.aim.2022.108829.

[AA7] Alejandro Cabrera and Miquel **Cueca**, *Dimensional reduction of Courant sigma models and Lie theory of Poisson groupoids*, Lett. Math. Phys. **112** (2022), no. 5, Paper No. 104, 48, DOI 10.1007/s11005-022-01596-1.

[AA8] Miquel **Cueca**, *The geometry of graded cotangent bundles*, J. Geom. Phys. **161** (2021), Paper No. 104055, 20, DOI 10.1016/j.geomphys.2020.104055.

[AA9] Miquel **Cueca** and Rajan Amit Mehta, *Courant cohomology, Cartan calculus, connections, curvature, characteristic classes*, Comm. Math. Phys. **381** (2021), no. 3, 1091–1113, DOI 10.1007/s00220-020-03894-y.

[AA10] Jialong **Deng**, *Sphere theorems with and without smoothing*, J. Geom. **113** (2022), no. 2, Paper No. 33, 15, DOI 10.1007/s00022-022-00647-1.

[AA11] ———, *Metric topology on the moduli space*, Appl. Gen. Topol. **22** (2021), no. 1, 11–15, DOI 10.4995/agt.2021.13066.

[AA12] ———, *Enlargeable length-structure and scalar curvatures*, Ann. Global Anal. Geom. **60** (2021), no. 2, 217–230, DOI 10.1007/s10455-021-09772-7.

[AA13] ———, *Curvature-dimension condition meets Gromov's n-volumic scalar curvature*, SIGMA Symmetry Integrability Geom. Methods Appl. **17** (2021), Paper No. 013, 20, DOI 10.3842/SIGMA.2021.013.

[AA14] Neslihan **Gügümcü** and L. H. Kauffman, *Parity, virtual closure and minimality of knotoids*, J. Knot Theory Ramifications **30** (2021), no. 11, Paper No. 2150076, 28, DOI 10.1142/S0218216521500760.

[AA15] Neslihan **Gügümcü**, Sam Nelson, and Natsumi Oyamaguchi, *Biquandle brackets and knotoids*, J. Knot Theory Ramifications **30** (2021), no. 9, Paper No. 2150064, 17, DOI 10.1142/S0218216521500644.

[AA16] Neslihan **Gügümcü** and Louis H. Kauffman, *Quantum invariants of knotoids*, Comm. Math. Phys. **387** (2021), no. 3, 1681–1728, DOI 10.1007/s00220-021-04081-3.

[AA17] Neslihan **Gügümcü** and Sofia Lambropoulou, *Braidoids*, Israel J. Math. **242** (2021), no. 2, 955–995, DOI 10.1007/s11856-021-2155-8.

[AA18] Roberta A. **Iseppi**, *The ghost fields and the BV extension for finite spectral triples*, J. Math. Phys. **63** (2022), no. 9, Paper No. 092302, 21, DOI 10.1063/5.0089795.

[AA19] ———, *A noncommutative geometric approach to the Batalin-Vilkovisky construction*, Quantum theory and symmetries, CRM Ser. Math. Phys., Springer, Cham, 2021, pp. 245–253.

[AA20] Claudia Garetto, Christian **Jäh**, and Michael Ruzhansky, *Hyperbolic systems with non-diagonalisable principal part and variable multiplicities, II: microlocal analysis*, J. Differential Equations **269** (2020), no. 10, 7881–7905, DOI 10.1016/j.jde.2020.05.038.

[AA21] Thorben **Kastenholz** and Mark Pedron, *The minimal genus of homology classes in a finite 2-complex*, Algebr. Geom. Topol. **22** (2022), no. 3, 1375–1415, DOI 10.2140/agt.2022.22.1375.

[AA22] Leyli Mammadova and Leonid **Ryvkin**, *On the extension problem for weak moment maps*, Homology Homotopy Appl. **24** (2022), no. 1, 143–159, DOI 10.4310/hha.2022.v24.n1.a8.

[AA23] Camille Laurent-Gengoux and Leonid **Ryvkin**, *The holonomy of a singular leaf*, Selecta Math. (N.S.) **28** (2022), no. 2, Paper No. 45, 38, DOI 10.1007/s00029-021-00753-z.

[AA24] Bas Janssens, Leonid **Ryvkin**, and Cornelia Vizman, *The L_∞-algebra of a symplectic manifold*, Pacific J. Math. **314** (2021), no. 1, 81–98, DOI 10.2140/pjm.2021.314.81.

[AA25] Camille Laurent-Gengoux and Leonid **Ryvkin**, *The neighborhood of a singular leaf*, J. Éc. polytech. Math. **8** (2021), 1037–1064, DOI 10.5802/jep.165 (English, with English and French summaries).

[AA26] Engelbert **Suchla**, *L^2-Invariants for Self-Similar CW-Complexes*, Doctoral dissertation, Universität Göttingen, 2016.

[AA27] Christopher **Wulff**, *Equivariant coarse (co-)homology theories*, SIGMA Symmetry Integrability Geom. Methods Appl. **18** (2022), Paper No. 057, 62, DOI 10.3842/SIGMA.2022.057.

[AA28] Alexander Engel, Christopher **Wulff**, and Rudolf **Zeidler**, *Slant products on the Higson-Roe exact sequence*, Ann. Inst. Fourier (Grenoble) **71** (2021), no. 3, 913–1021, DOI 10.5802/aif.3406 (English, with English and French summaries).

[AA29] Christopher **Wulff**, *Secondary cup and cap products in coarse geometry*, Res. Math. Sci. **8** (2021), no. 3, Paper No. 36, 64, DOI 10.1007/s40687-021-00273-4.

[AA30] Christopher **Wulff** and Alexander Engel, *Coronas for properly combable spaces*, J. Topol. Anal. (2021). online ready.

[AA31] Vito Felice **Zenobi**, *The adiabatic groupoid and the Higson-Roe exact sequence*, J. Noncommut. Geom. **15** (2021), no. 3, 797–827, DOI 10.4171/jncg/422.

[AA32] Thomas **Schick** and Vito Felice **Zenobi**, *Positive scalar curvature due to the cokernel of the classifying map*, SIGMA Symmetry Integrability Geom. Methods Appl. **16** (2020), Paper No. 129, 12, DOI 10.3842/SIGMA.2020.129.

[AA33] Vito Felice **Zenobi**, *Adiabatic groupoid and secondary invariants in K-theory*, Adv. Math. **347** (2019), 940–1001, DOI 10.1016/j.aim.2019.03.003.

2.1.4 Publications available as preprint of doctoral and postdoctoral researchers who received funding from other sources

[PA1] Simone **Cecchini**, Mehran Seyedhosseini, and Vito **Zenobi**, *Relative torsion and bordism classes of positive scalar curvature metrics on manifolds with boundary*, 2022. arXiv:2009.13718.

[PA2] Simone **Cecchini**, Daniel Räde, and Rudolf **Zeidler**, *Nonnegative scalar curvature on manifolds with at least two ends*, 2022. arXiv:2205.12174.

[PA3] Simone **Cecchini**, Bernhard Hanke, and Thomas Schick, *Lipschitz rigidity for scalar curvature*, 2022. arXiv:2206.11796.

[PA4] Simone **Cecchini** and Rudolf **Zeidler**, *The positive mass theorem and distance estimates in the spin setting*, 2021. arXiv:2108.11972.

[PA5] Tom **Dove**, *Local formulations of T-duality for principal torus bundles*, 2021. arXiv:2104.05984.

[PA6] Tom **Dove**, Thomas **Schick**, and Mario Velasquez, *A fixed point decomposition of twisted equivariant K-theory*, 2022. arXiv:2202.055.588.

[PA7] Neslihan **Gügümcü**, Louis H. Kauffman, and B. Gabrovsek, *Invariants of bonded knotoids and applications to protein folding*, 2022. Invited paper, to appear in Symmetry, Special Issue: Topological Methods in Chemistry and Molecular Biology, 2022.

[PA8] Neslihan **Gügümcü** and B. Gabrovsek, *Invariants of multi-linkoids*, 2022. arXiv:2204.11234.

[PA9] Neslihan **Gügümcü** and Ahmet Batal, *K-color region select game*, 2022. arXiv:2205.03200.

[PA10] Thorben **Kastenholz**, Jean-Pierre Mutanguha, and Rachel Boyd, *The minimal genus problem for right angled Artin groups*, 2021. arXiv:2108.02914; positively reviewed for Geom.Dedicata.

[PA11] Thorben **Kastenholz** and Jens Reinhold, *Simplicial volume and essentiality of manifolds fibered over spheres*, 2021. arXiv:2107.05892; positively reviewed for J. of Topology.

[PA12] Oscar Cosserat, Camille Laurent-Gengoux, Alexei Kotov, Leonid **Ryvkin**, and Vladimir Salnivko, *On Dirac structures admitting a variational approach*, 2021. arXiv:2109.00313.

[PA13] Leonid **Ryvkin**, Casey Blacker, and Antonio Miti, *Reduction of L-infinity-algebras of observables on multisymplectic manifolds*, 2022. arXiv:2206.03137.

[PA14] Jonathan **Taylor**, *Localised Hilbert modules and weak noncommutative Cartan pairs*, 2021. arxiv.org/abs/2110.14390.

[PA15] ______, *Essential commutative Cartan subalgebras of C^*-algebras*, 2022. arxiv.org/abs/2206.09634.

[PA16] Jonathan **Taylor**, Ralf **Meyer**, and Ali I. Raad, *Inductive Limits of Noncommutative Cartan Inclusions*, 2022. arxiv.org/abs/2205.14053.

[PA17] Christopher **Wulff**, *Generalized asymptotic algebras and E-theory for non-seperable C*-algebras*, 2022. arXiv:2212.07216.

2.2 10 most important publications of RTG as a whole

[ET1] Dorothea **Bahns**, Klaus Fredenhagen, and Kasia Rejzner, *Local nets of von Neumann algebras in the sine-Gordon model*, Comm. Math. Phys. **383** (2021), no. 1, 1–33, DOI 10.1007/s00220-021-03961-y.

[ET2] Léo **Bénard**, Jérôme Dubois, Michael Heusener, and Joan Porti, *Asymptotics of twisted Alexander polynomials and hyperbolic volume*, Indiana Univ. Math. J. **71** (2022), no. 3, 1155–1207.

[ET3] Jörg **Brüdern** and Trevor Wooley, *On Waring's problem for larger powers*, 2022. arXiv:2211.10380.

[ET4] Simone **Cecchini** and Rudolf **Zeidler**, *Scalar and mean curvature comparison via the Dirac operator*, Geometry & Topology. to appear, compare G&T list of upcoming articles: https://msp.org/soon/coming.php?jpath=gt.

[ET5] Miquel **Cueca**, Chenchang **Zhu**, and Florian **Dorsch**, *Shifted symplectic higher Lie groupoids and classifying spaces. With an appenxid by Florian Dorsch*, Adv. Math. **413** (2023), Paper No. 108829, DOI 10.1016/j.aim.2022.108829.

[ET6] Paolo Piazza, Thomas **Schick**, and Vito Felice **Zenobi**, *Mapping analytic surgery to homology, higher rho numbers and metrics of positive scalar curvature*. arXiv:1905.11861v2, current version submitted to Memoirs of the AMS.

[ET7] Bartosz Kosma Kwaśniewski and Ralf **Meyer**, *Essential crossed products by inverse semigroup actions: Simplicity and pure infiniteness*, Doc. Math. **26** (2021), 271–335, DOI 10.25537/dm.2021v26.271-335.

[ET8] Anton Savin and Elmar **Schrohe**, *An index formula for groups of isometric linear canonical transformations*, Doc. Math. **27** (2022), 983–1013.

[ET9] ———, *Local index formulae on noncommutative orbifolds and equivariant zeta functions for the affine metaplectic group*. part A, Adv. Math. **409** (2022), no. part A, Paper No. 108624, 37, DOI 10.1016/j.aim.2022.108624.

[ET10] Damaris **Schindler** and Shuntaro Yamagishi, *Density of rational points near/on compact manifolds with certain curvature conditions*, Adv. Math. **403** (2022), Paper No. 108358, 36, DOI 10.1016/j.aim.2022.108358.