

DECEMBER 16–18, 2019

DANIELE ANGELLA

mini-course

COHOMOLOGICAL AND METRIC ASPECTS IN COMPLEX NON-KÄHLER GEOMETRY

Kähler geometry provides analytic techniques to extend results from the algebraic to the transcendental setting. Non-Kähler geometry is then the attempt to perform a separate analysis of the complex and symplectic contributions. Cohomological invariants and canonical Hermitian metrics are useful tools for a tentative classification of compact complex manifolds.

First lecture. Cohomological invariants of complex manifolds.

- Complex and almost-complex manifolds. [Huy05]
- The double complex of forms on a complex manifold. [Ste18c]
- Cohomologies of complex manifolds. [Huy05, Sch07]
- Cohomologies of quotients of Lie groups. [Rol11]

Second lecture. Cohomological decompositions.

- Cohomological decomposition on complex manifolds: the $\partial\bar{\partial}$ -Lemma. [DGMS75, AT13]
- Behaviour of the $\partial\bar{\partial}$ -Lemma under deformations and modifications. [AT13, ASTT17, RYY17, Ste18b]
- Generalized complex geometry. [Gua04]
- Cohomological decompositions on symplectic manifolds. [Cav05, AT15]

Third lecture. Special Hermitian metrics on complex manifolds.

- Hermitian and Kähler metrics on complex manifolds. [Huy05]
- Hermitian connections. [Gau97]
- The Chern connection, and constant Chern-scalar curvature metrics in conformal classes. [ACS17]

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(Daniele Angella) DIPARTIMENTO DI MATEMATICA E INFORMATICA “ULISSE DINI”, UNIVERSITÀ DEGLI STUDI DI FIRENZE,
 VIALE MORGAGNI 67/A, 50134 FIRENZE, ITALY
 Email address: daniele.angella@gmail.com
 Email address: daniele.angella@unifi.it
 URL: <http://sites.google.com/site/danieleangella/>